Chapter 3

No Persistent Pulsations in Aql X-1 as it Fades into Quiescence

Abstract

We searched for coherent X-ray pulsations from Aql X-1 in a series of RXTE observations taken shortly after a recent outburst. During the course of these observations, Aql X-1 passes through an apparent “propeller” phase as its luminosity fades to its quiescent value. No pulsations were detected, and we place upper limits (ranging from 0.52% to 9.0%) on the fractional RMS amplitude of any periodic signal contained in the various data sets searched. This result has implications for the geometry of the system, if the quiescent luminosity is due to continued low-level accretion. Alternatively, our result supports the idea that the quiescent luminosity may be due to thermal emission.

3.1 Introduction

The origin of the quiescent emission observed from low-magnetic field transient neutron stars (NSs) has posed something of a puzzle. When first discovered observationally (in the transient Cen X-4; van Paradijs et al., 1987), the X-ray emission, fit with a blackbody spectrum, was too faint to have originated over the entire surface of a 10 km NS. It was suggested that the emission was due to accretion over a fraction (\(~\text{few}\%\)) of the NS surface. As additional quiescent NS X-ray spectra were observed and fit with a blackbody spectrum (Verbunt et al., 1994; Asai et al., 1996a,b), the small implied emission areas seemed to confirm this view.

The cause of accretion anisotropy over the surface of a low B-field NS has been purported to be the “propeller effect” (Illarionov & Sunyaev, 1975; Stella et al., 1986), in which at low accretion rates, the magnetic field increases in importance relative to the gravitational field in determining the accretion geometry, perhaps expelling much of the accretion from the system when the effect of the magnetic field is comparable to or stronger than that of the gravitational field at the radius where the keplerian frequency is equal to the spin frequency of the NS. When this occurs, some fraction of the accretion may follow the magnetic field lines to the magnetic poles which, if these are offset from the rotational poles, could conceivably produce X-ray pulsations. The efficiency of such a propeller has been estimated recently (Menou et al., 1999). However, some observations of decreasing torque with increasing luminosity may indicate deviations from this standard magnetic accretion scenario in low mass X-ray binaries (Chakrabarty et al., 1997). If, after an outburst, accretion continues into quiescence then pulsations might be expected, particularly if the NS can be shown to have gone through a “propeller phase,” in which the accretion geometry in the vicinity of the NS is dominated by the magnetic field, rather than by the gravitational field. No pulsations from these objects in quiescence have yet been reported, although they have been searched for (Verbunt et al., 1994; Asai et al., 1996b).

Thus, until recently, the view of the quiescent luminosity of low-B transient NSs
has been as due to continued accretion over a fraction of the surface of the NS, perhaps caused by modified accretion geometry due to the effects of the magnetic field, which may therefore give rise to pulsations in the X-ray intensity.

A different view of the origin of the quiescent luminosity has recently been described (Brown et al., 1998), in which nuclear reactions at the base of the NS crust keep the NS core heated to temperatures \((\approx 10^8 \text{ K})\) sufficient to explain a large fraction, if not all, of the quiescent luminosity of these objects \((10^{32-33} \text{ erg s}^{-1})\); in this manner, the quiescent luminosity can be produced in the absence of active accretion. The discrepancy in the emission area can be explained as due to an incorrect blackbody assumption – specifically, if accretion is shut off, metals stratify in the NS atmosphere (Alcock & Illarionov, 1980; Romani, 1987), producing a pure H photosphere, in which the free-free opacity, which decreases with increasing frequency, dominates, permitting higher E photons to originate at greater depth, thus at higher temperatures (Rajagopal & Romani, 1996; Zavlin et al., 1996). This produces an emergent spectrum which is spectrally harder than a simple blackbody, and – when described as a simple blackbody – requires a higher \(T\), and subsequently lower emission area, to parameterize it. Spectral fits with H atmosphere spectra show that the observed X-ray spectra from the three such objects for which data exists (Cen X-4, Aql X-1, and 4U 1608-522) imply emission areas consistent with a 10 km NS (Rutledge et al., 1999). Thus, both their quiescent luminosities and their emergent X-ray spectra can be explained in this manner, without invoking active accretion during quiescence. An attractive feature of this explanation is that it accounts for the similar luminosities of these three objects, without requiring their quiescent mass accretion rates (a product of the propeller efficiency and mass accretion rate through the accretion disk) to be serendipitously similar.

It remains an open question if accretion (at a level of \(\dot{M} \lesssim 10^{-12} M_\odot \text{ yr}^{-1}\)) occurs at all, contributing some fraction of the quiescent X-ray luminosity. There are few observational means by which this may be investigated. One such possibility is to search for metal lines in the quiescent X-ray spectrum, which, if present, would imply that metals are being fed into the atmosphere at a rate greater than gravity can
stratify it (Bildsten et al., 1992; see Zavlin et al., 1996 for X-ray spectra of metallic atmospheres). Such spectra can be investigated by the latest generation of X-ray spectroscopy missions (AXAF and XMM). A second method is to search for intensity variability during quiescence. In particular, as described above, if pulsational variability at a particular frequency were to be found in quiescence, this would support the scenario of active accretion along the magnetic field lines to the NS surface. Alternatively, if the NS magnetic field were strong enough to significantly affect the photospheric opacity (> 10^{10} G; cf. Zavlin et al., 1995), this would produce apparent “hot spots” near the magnetic poles of the NS which, if offset from the rotation axis, could give rise to pulsations; however, it is thought that type-I X-ray bursting sources have magnetic fields below this value. Clearly discovery of pulsations or constraints on the pulsed fraction during X-ray quiescence (following the onset of a “propeller”) would provide useful limits for models of quiescent NS emission.

The system Aql X-1 lends itself to such a search for pulsations. During the X-ray decline of a recent outburst, the intensity at first decreased by one order of magnitude over 17 days, then abruptly (beginning at \( L_x \approx 10^{36} \text{ erg s}^{-1} \)) by 3 orders of magnitude over 3 days, which was coupled with the sudden spectral hardening of the X-ray intensity (Campana et al., 1998; Zhang et al., 1998a). The X-ray intensity stopped dropping at a luminosity \( L_x \sim 10^{33} \text{ erg s}^{-1} \), comparable to the quiescent luminosity observed previously (Verbunt et al., 1994). This was interpreted as evidence for a propeller-phase, though this remains to be confirmed through repeated observations of Aql X-1 in outburst. If indeed this behavior does indicate a propeller phase in Aql X-1, it offers the opportunity to search for pulsations during the period when the NS magnetic field substantially alters the accretion geometry.

Here we have searched for, and not found, pulsations in Aql X-1 during and immediately following its supposed propeller phase. In section 3.2, we describe our search method and the results of our analysis. In section 3.3 we discuss these results and our conclusions.
3.2 Analysis

3.2.1 Search Methodology

We used high time resolution data taken with RXTE/PCA (Swank et al., 1996). The PCA detectors have a total geometric area of 6500 cm², and a nominal energy range of 2-60 keV. We selected for analysis four observations, based on the results of Zhang et al. (1998b). These were listed by Zhang et al. as observations 8, 10, 11, plus a later observation, 12, which was not analyzed by Zhang due to the faintness of the source – which made their spectral analysis infeasible, but can still be useful in our search for pulsations. Based on Zhang’s results, the beginning of the propeller-phase appears to be between observations 9 and 10, and so our timing analyses of observations 10-12 are expected to reveal pulsations created by the magnetic-field modified accretion geometry (if any), while observation 8 is included simply for comparison purposes while the source was moderately brighter. We list details of the observations in Table 3.1. Data were obtained from the XTE archive at HEASARC.

For our data analysis, we used the standard FTOOLS/XTE v4.0. Using standard-2 formatted data, we extracted the X-ray spectrum from each observation and subtracted background counts (estimated using pcabackest tool), and examined the counts spectra. Based on the spectrum of the faintest observation (number 12), which becomes background dominated above ~12keV, we selected for our analysis PHA bins 0-20, corresponding in energy roughly to 2-12 keV. We then extracted individual photon events from data in E125 mode (which has time resolution of ~125μs), and corrected the photon arrival times (time of arrival; TOAs) to the solar system.

<table>
<thead>
<tr>
<th>Observation</th>
<th>Start Time / End Time (UT)</th>
<th>Start Time (JD)</th>
<th>Start Time (MET)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20098-03-08-00</td>
<td>1997-03-01 21:33 1997-03-02 00:21</td>
<td>2450509.398</td>
<td>99869572</td>
</tr>
<tr>
<td>20098-03-10-00</td>
<td>1997-03-05 22:55 1997-03-06 01:28</td>
<td>2450513.456</td>
<td>10022027</td>
</tr>
<tr>
<td>20098-03-11-00</td>
<td>1997-03-08 21:31 1997-03-09 01:11</td>
<td>2450516.394</td>
<td>100474022</td>
</tr>
<tr>
<td>20098-03-12-00</td>
<td>1997-03-10 21:25 1997-03-10 23:55</td>
<td>2450518.389</td>
<td>100646431</td>
</tr>
</tbody>
</table>
barycenter.

Zhang et al. (1998b) reported strong evidence for a 549 Hz pulsation frequency, observed during \( \sim 10 \) seconds following a type-I X-ray burst from Aql X-1. (It is possible that this may be the second harmonic of the NS spin frequency.) Similar pulsations have been observed from type-I bursts of 5 other sources (1728-34, Strohmayer et al., 1996; 1636-53, Zhang et al., 1996; 1731-260, Smith et al., 1997; a Galactic center source, possibly MXB 1743-29, Strohmayer et al., 1997; and 1702-43, Markwardt et al., 1999). Rather than restrict our efforts to a small range of frequencies around the previously measured value, we chose to do a more general search for pulsations over the broad frequency range 0.5 – 1024 Hz. This range includes all previously reported rotation frequencies for type-I X-ray bursters. We were able to do this more general search without a significant loss of sensitivity for reasons which we explain below, in section 3.2.4.

The Aql X-1 system consists of a NS and a companion star, both orbiting the system’s center of mass with a period of about 19 hours (see section 3.2.2 below). As a result of this orbital motion, the NS is accelerating along our line of sight. Any pulsations will therefore be observed with a doppler shifted frequency which is not constant in time. In a standard fast Fourier transform (FFT) analysis, the spectral power resulting from such a signal may be spread out over many frequency bins, drastically decreasing the probability of detection. For example, consider a 550 Hz pulsar in a 19 hour circular orbit with a maximum projected velocity of 100 km s\(^{-1}\). If we calculate a coherent power spectrum using \( T = 2048 \) seconds of data, the power from this pulsar will be spread over as many as 70 independent frequency bins (for the worst case orbital phase, the observed pulsar frequency covers a range \( \Delta f = 0.034 \text{ Hz} \), giving \( T\Delta f = 70 \) bins). To counteract this effect, we attempt to remove the acceleration of the pulsar signal in the time domain before performing the FFT. Because we do not have complete knowledge of Aql X-1’s orbital parameters, we must cover the possible orbital parameter space with a number of acceleration trials and repeat the Fourier analysis for each.

For our purposes, the orbit (assumed circular) can be characterized by three in-
dependent parameters — the orbital period $P_{\text{orb}}$, the projected circular velocity $v$, and the orbital phase $x_0$ of the NS at the start of the observation in question. The projected circular velocity is the magnitude of the NS’s circular velocity projected along our line of sight, $v = v_{NS} \sin i$, where $i$ is the angle between the line of sight and the normal to the plane of the orbit. Note that we could equally well have chosen the projected orbital radius $(a \sin i = vP_{\text{orb}}/2\pi)$ in lieu of this parameter. The orbital phase $x_0$ is measured in cycles, and is therefore in the range $[0, 1]$, with $x_0 = 0$ corresponding to longitude $= 0$ (where the observed pulsar frequency would be measured at its minimum). The signal from a pulsar in such an orbit would be observed with a doppler shifted frequency

$$f(t) = f_0 \left[ 1 - \frac{v}{c} \cos \left( \frac{2\pi}{P_{\text{orb}}} t + 2\pi x_0 \right) \right], \quad (3.1)$$

where $f_0$ is the rest frequency of the pulsar, $c$ is the speed of light, and $t$ is the elapsed time since phase $x_0$.

Pulsar orbital acceleration searches are typically carried out (see, for example, Anderson et al., 1990) by correcting data with an assumed constant acceleration $f(t) = f_0 + \dot{f} t$. This is applicable when the data cover only a small part of the pulsar’s orbit, or the pulsar is very strong. In the first case, the short span of orbit is well approximated by a constant acceleration, while in the second case, some spreading of the feature in the power spectrum can be tolerated without the signal disappearing into the noise. For the Aql X-1 searches reported here, we assumed that neither of these conditions was satisfied. Our acceleration searches therefore fully correct for an assumed circular orbit $(P_{\text{orb}}, v, x_0)$. As compared with the constant acceleration method, this circular acceleration method may require more trial accelerations to cover a given orbital phase space, but the detection significance is greatly increased. This is because the circular method recovers significantly more signal power in a single power spectrum bin, more than making up for the increased number of trials (the detection significance is exponential in recovered power, but only linear in the number of trials).
After preparing an energy selected, barycentered photon TOA list, we proceeded with the search method as follows. First, we assumed particular values for the orbital parameters (i.e., a trial acceleration) from within the search phase space (of orbital period, velocity, and initial phase; see Table 3.2). We corrected the TOAs for the assumed acceleration by introducing a corrected time $\tilde{t}$, which is a function of the original time $t$, such that the frequency as a function of $\tilde{t}$ is constant. Equivalently, we require the integrated phase to be linear in $\tilde{t}$:

$$x(t) - x_0 = \int_0^t f(t')dt' = f_0 \tilde{t}. \quad (3.2)$$

Integrating Equation (3.1) for a given acceleration trial $(P_{\text{orb}}, v, x_0)$, we see that the $i^{th}$ TOA $t_i$ is corrected to:

$$\tilde{t}_i = t_i + \frac{\beta}{\omega}\left[\sin(2\pi x_0) - \sin(\omega t_i + 2\pi x_0)\right], \quad (3.3)$$

where $\beta = v/c$ and $\omega = 2\pi/P_{\text{orb}}$.

These corrected TOAs were used to construct a time series which was then FFTed and used to produce an estimate of the power density spectrum (PDS). The PDS was searched for candidates (frequency bins containing statistically significant excess power). The process was repeated for each acceleration trial. The spacing of acceleration trials in (orbital period, velocity, and phase)-parameter space was chosen to allow sensitivity to as weak a signal as possible while keeping the computational requirements reasonable.

### 3.2.2 Determination of Searched Parameter Space

Similar, but inconsistent, values for the orbital period of Aql X-1 have been published by two groups. Based on observations of Aql X-1 during outburst, Chevalier & Ilovaisky (1991) measured the orbital period to be $18.97\pm0.02$ hr. Shahbaz et al. (1998) determined the orbital period in quiescence to be $19.30\pm0.05$ hr. In a more recent analysis, Chevalier & Ilovaisky (1998) report the period to be $18.9479\pm0.0002$ hr,
again measured during outburst, but they also report a quiescent period within 0.02% of this value. To be safe, we chose to cover a range of orbital periods that encompasses a $3\sigma$ range in all of these measurements (as well as the periods in between):

$$18.91 < P_{\text{orb}} < 19.45 \text{ hr.} \quad (3.4)$$

Fortunately, we were able to cover this entire range with one trial value of $P_{\text{orb}}$ (see section 3.2.3 below).

The projected circular velocity $v$ has not been measured. We can determine an upper limit on $v$ for our search by assuming a value for the NS mass, $m_{\text{NS}} = 1.4 \, M_\odot$ ($M_\odot$ = one solar mass), and choosing a maximum companion mass to which we will be sensitive. We can then use our knowledge of the orbital period to calculate the orbital velocity of the NS. Recently, the true optical counterpart of Aql X-1 has been identified as a late K type star (Chevalier et al., 1999). Again, to be safe we decided to cover a range of velocities corresponding to a companion star mass as high as $1.0 \, M_\odot$ (for all possible orbital inclination angles). This results in a search range of

$$0 \leq v \leq 130 \text{ km s}^{-1}. \quad (3.5)$$

Note that our search was also sensitive to larger companion masses in a restricted range of inclination angles; e.g. our search covered companion masses up to $2.0 \, M_\odot$ for $0^\circ \leq i \leq 40^\circ$.

For the starting orbital phase for each observation, rather than rely on a particular ephemeris, we simply chose to search the full range

$$0 \leq x_0 \leq 1 \text{ cycle.} \quad (3.6)$$

Our search phase space is summarized in Table 3.2.
Table 3.2: Aql X-1 searched parameter space

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{orb}}$</td>
<td>18.91 – 19.45 hr</td>
</tr>
<tr>
<td>$v_{\text{NS}} \sin i$</td>
<td>0 – 130 km s$^{-1}$</td>
</tr>
<tr>
<td>Initial Orbital Phase</td>
<td>0 – 1 cycles</td>
</tr>
<tr>
<td>Pulsar Spin Frequency</td>
<td>0.5 – 1024 Hz</td>
</tr>
</tbody>
</table>

3.2.3 Searches Performed

The *RXTE* observations included in this analysis were roughly 9 – 13 kiloseconds in duration. The TOA data in each observation are broken up into two or three cohesive sections, separated by gaps due to earth occultations. Each continuous section is at least 2048 seconds long, and we chose to study these continuous blocks individually. Observation 10, with one occultation drop-out, is therefore divided into two sections, which we call observations 10 a and 10 b. Observations 11 and 12 are similarly divided (into 11 a, 11 b, 11 c, 12 a, and 12 b). We analyzed only the first section of observation 8, for a total of eight separate data sets. The photons from a given data set were binned into a $2^{23}$ point time series, with a time resolution of $\Delta t = 244 \mu\text{sec}$. Over the course of the observations, the source count rate decreases from about 670 counts s$^{-1}$ at the beginning of observation 8 to less than 10 counts s$^{-1}$ by the end of observation 12, with a background rate of about 31 – 37 counts s$^{-1}$ (see Table 3.3).

Our search method was described above in section 3.2.1. We now describe in detail the method used to determine the grid of trial values for the orbital parameters. Since we must discretize a continuous phase space, there will likely be some offset between a signal’s actual parameters and our nearest trial values. The effect of this offset will be some spreading out of the signal power in the power spectrum, since the time dependence of the signal’s frequency will not have been completely removed. In choosing the orbital trials, the general idea was to space them just finely enough to keep the remaining frequency drift within tolerable limits (to be quantified below).
We began analytically, and then made some empirical adjustments. Considering each of the three orbital parameters \( (P_{\text{orb}}, v, x_0) \) separately, we took partial derivatives of Equation (3.1), to determine the frequency drift that results from offsets in the individual parameters. For example, an error in the velocity \( (\partial v = c \partial \beta) \) causes a drift of

\[
\partial f = f_0 \cos (\omega t + 2\pi x_0) \, \partial \beta.
\]  

(3.7)

We are interested only in the extrema of the frequency range covered by the signal over the course of the observation time \( T \), and for simplicity, we chose to make our velocity trial values independent of the other parameters. We therefore replace the cosine factor in Equation (3.7) by its maximum change over the course of an observation \( (T = 2048 \, \text{s}) \), and we replace \( f_0 \) by our maximum search frequency \( (f_0 \to 1024 \, \text{Hz}) \). The maximum frequency drift caused by a velocity offset is therefore

\[
\partial f_{\text{max}} = 1024 \, 0.19 \, \partial \beta \, \text{(Hz)}. 
\]  

(3.8)

A “tolerable” drift must be no greater than the independent frequency resolution of the power spectrum, \( F \). Since we are considering each orbital parameter separately, for these initial calculations we restrict the error-induced drift from each to \( F/2 \). This is somewhat arbitrary, and the actual spacings used were decided on with the help of simulations. Note that the allowable spacing of the trials can be twice the maximum allowable offset. We now have for the velocity trial spacing

\[
\Delta v_{\text{trial}} \lesssim \frac{cF}{1024 \, 0.19} = 0.77 \left( \frac{F}{5 \times 10^{-4} \, \text{Hz}} \right) \, \text{km/s}^{-1}. 
\]

(3.9)

For a coherent FFT of 2048 seconds of data, the independent Fourier resolution is \( F = 1/T = 4.88 \times 10^{-4} \, \text{Hz} \). The search strategy that we settled on actually used a larger frequency resolution (see below); for now we will leave the expressions for the trial spacings in terms of \( F \) explicitly.

Again, for simplicity, we chose to keep the spacing of the \( P_{\text{orb}} \) and \( x_0 \) trials independent of \( P_{\text{orb}} \) and \( x_0 \), but we did allow for velocity dependence. For the orbital
period, we ultimately find

$$\Delta P_{\text{trial}} \lesssim 2800 \left( \frac{F}{5 \times 10^{-4} \text{ Hz}} \right) \left( \frac{v}{100 \text{ km s}^{-1}} \right)^{-1} \text{ seconds.} \quad (3.10)$$

Even for the smallest Fourier resolution ($F = 4.88 \times 10^{-4}$ Hz) and the largest velocity in our search range ($v = 130 \text{ km s}^{-1}$), the orbital period trial spacing turns out to be larger than our target search range of $(19.45 \text{ hr} - 18.91 \text{ hr}) 3600 \text{ s} \text{ hr}^{-1} = 1944 \text{ s}$. In other words, a single trial value was sufficient to cover our entire search range in $P_{\text{orb}}$. Thus, we did not actually search over trial values of the orbital period. For the initial orbital phase trial spacing, we find

$$\Delta x_{\text{trial}} \lesssim 1.2 \times 10^{-3} \left( \frac{F}{5 \times 10^{-4} \text{ Hz}} \right) \left( \frac{v}{100 \text{ km s}^{-1}} \right)^{-1} \text{ cycles.} \quad (3.11)$$

If we were to use these spacings with coherent 2048 second FFTs, each of the eight data sets would require over 95,000 trial accelerations. This would require $\gtrsim 100$ days of CPU time on a Sun Ultra 10 workstation. To reduce the computational requirements, we chose to utilize incoherently stacked power spectra. The original data of duration $T$ are divided into $S$ sections of duration $T/S$. Each section is FFTed individually and the $S$ individual power spectra are (incoherently) added together. The independent Fourier step size has been increased by a factor of $S$ ($F = 1/(T/S) = S/T$) and the trial parameter spacings increase by the same factor. The total number of trials is therefore reduced by a factor of $S^2$ (since we are searching over two parameters). The reduction in the number of trials comes at the cost of reduced sensitivity, so the number of stacks $S$ should be kept as small as possible, just barely bringing the number of trials to a feasible level. For our analyses, we chose to use $S = 4$.

With the above analytical calculations as guides, we used simulations to determine the trial spacings actually used in the search. Our simulations upheld the decision to use a single $P_{\text{orb}}$ trial. Since there are indications that the orbital period is more likely to lie towards the lower end of our search range $(18.91 - 19.45 \text{ hr})$, we chose to
use $P_{\text{orb}} = 19.00 \text{ hr}$ in our trials. We decided to use a velocity trial spacing of

$$\Delta v_{\text{trial}} = 2.0 \text{ km s}^{-1}.$$  

(3.12)

This is about $1.5 \times$ finer than the spacing calculated above (recall that we are using $S = 4$ stacks). For the orbital phase, we decided on a trial spacing of

$$\Delta x_{\text{trial}} = 0.011 \left( \frac{v}{100 \text{ km s}^{-1}} \right)^{-1} \text{ cycles},$$

(3.13)

which is about $2.3 \times$ coarser than the spacing indicated in the analytical calculation. For small velocities, we used a minimum of 16 $x_0$ trials (except for the single $v = 0$ trial).

Using these spacings with $2 \times$ oversampled power spectra (exactly like our actual searches), we determined that at least 98% of the time, we were able to recover at least 77% of a simulated signal’s power in a single spectral bin, even in the least favorable regions of our search phase space. We simulated signals whose orbital parameters were offset from our search trials and whose pulsation frequencies were offset from our discrete Fourier frequencies. The reduction in recovered signal power is due to a combination of these factors. (Note that it is mere coincidence that a simple FFT recovers, on average, 77% of a signal’s power based solely on offset from the discrete Fourier frequencies. In a $2 \times$ oversampled spectrum, the $\textit{minimum}$ power recovered, based solely on frequency offset, is 81%.) Our peak detected power was not always in the frequency bin closest to the rest frequency of the simulated pulsar. This has no effect on the detection of pulsations; the true frequency and orbit of the pulsar can be refined after the initial detection.

In total, we covered the orbital phase space with 4069 acceleration trials. The same accelerations were used for each of the eight data sets.
3.2.4 Estimation of Detection Sensitivities

To characterize the sensitivity of our search, we wish to place quantitative limits on the minimum signal strength required for a significant detection. Since the noise statistics of our power spectra are well understood, it is a simple matter to determine the detection threshold, the minimum spectral power $P_{\text{det}}$ required for a detection. This information can be used to determine the minimum required signal strength. For a review of detection thresholds, detection sensitivities, and upper limits, see Vaughan et al. (1994).

Our stacked power spectra were constructed by summing four individual spectra, each normalized to a mean power of one (note that this convention differs from the commonly used Leahy normalization (Leahy et al., 1983), for which the mean noise power is two). In the absence of a signal, the power $P$ in a given spectral bin follows a $\chi^2$ distribution. Specifically, $2P$ is $\chi^2$ distributed with $2S = 8$ degrees of freedom. The probability that the power $P$ in a single PDS bin will exceed a given value $P_0$ is therefore

$$p(P > P_0) = e^{-P_0} \left(1 + P_0 + \frac{1}{2} P_0^2 + \frac{1}{6} P_0^3 \right).$$

(3.14)

Considering each frequency bin of each PDS to be one search “trial,” our entire search consisted of $N_{\text{trials}} = 3.4 \times 10^{10}$ such trials. This is the product of the number of observations searched (8), the number of acceleration trials for each observation (4069), and the number of frequencies searched in each power spectrum ($2^{30}$). The latter number is equal to the frequency range searched (1024 Hz) divided by the independent frequency resolution ($F = (S/2048) \text{ Hz}$) times the oversampling factor (2). Of course, neighboring bins in an oversampled spectrum are not independent, and neighboring acceleration trials may not produce truly independent spectra. Thus $N_{\text{trials}}$ is an upper limit to the number of statistically independent trials in the search. Since we are overestimating the number of independent trials, our detection threshold will be conservative.

The statistical significance $S$ of a measured power $P_0$ is equal to the probability that the power was produced by a random noise fluctuation. For small $p$ (large
powers), the significance is
\[ S = N_{\text{trials}} \, p(P > P_0). \]  
\[ (3.15) \]

To achieve our target significance of \(10^{-4}\), we require a PDS power of at least
\[ P_{\text{det}} = 43. \]  
\[ (3.16) \]

For a given data set containing \(N_t\) total counts, \(N_s\) of which were emitted by the source, we can relate the source strength to the expected spectral power by
\[ \langle P \rangle = 4 + \frac{1}{4} \frac{N_s^2}{N_t} \mathcal{F}^2 \]  
\[ (3.17) \]

(Buccheri et al., 1987; van der Klis, 1989), where \(\mathcal{F}\) is the pulsed fraction, i.e. the fraction of source counts that actually contribute to the pulsation. The first term in Equation (3.17) is the expected noise power (\(\langle P_N \rangle = 4\) because we have summed four power spectra each normalized to unity), while the second term is the expected signal power. Here, we have assumed that the signal waveform is sinusoidal. \(N_s\) and \(N_t\) are determined for each data set before beginning the pulsation search. Therefore, the sensitivity of each search is determined as a limit on the pulsed fraction \(\mathcal{F}\).

We now calculate the detection sensitivities for the eight data sets searched. For a given observation, the detection sensitivity is expressed as the minimum pulsed fraction required to produce a spectral power exceeding \(P_{\text{det}}\) with high confidence. Instead of simply solving Equation (3.17) for \(\mathcal{F}\) using the detection threshold power \(P_{\text{det}} = 43\), we must allow for statistical variation of the spectral power produced by a source of a given strength, as well as variation in the recovered power due to the discrete nature of the search trials.

To determine the detection sensitivity, we must consider the probability distribution of power in a spectral bin containing a signal plus noise. If, in the absence of noise, the signal produces a power \(P_{\text{sig}}\), then the power \(P\) in the presence of noise will
be distributed according to

\[ p_n(P > P_0; P_{\text{sig}}) = \exp[-(P_0 + P_{\text{sig}})] \sum_{m=0}^{\infty} \sum_{k=0}^{m+n-1} P_0^k P_{\text{sig}}^m / (k!m!) \]  

(3.18)

(Groth, 1975), where \( n \) is the number of independent bins that were summed to produce \( P \) (i.e., in our case \( n = S = 4 \)). Thus, \( p_n(P > P_0; P_{\text{sig}}) \) is the probability that the power will exceed \( P_0 \) in the spectral bin containing the signal. It is important to note that noise power and signal power are not simply additive; \( P \) will not always exceed \( P_{\text{sig}} \) (see Vaughan et al. (1994) for a discussion of this point). For a given \( N_s \) and \( N_t \), we would like to find the pulsed fraction \( \mathcal{F} \) that is 95% likely to produce power \( P > P_{\text{det}} \).

To ensure that our reported sensitivity is as conservative as possible, we will not naively invert Equation 3.18 for \( P_{\text{sig}} \) with \( P_0 = P_{\text{det}} \). Instead we consider the worst-case scenario covered by our search parameter space. Due to the discrete binning of photons in the construction of the time series, the recovered signal power in the FFT falls off with increasing frequency. A signal near our maximum search frequency — 1024 Hz, which is half the Nyquist frequency of our power spectra — will produce 81% of the power that would be recovered from a low frequency signal of the same intrinsic strength. Also, due to our discrete grid of acceleration trials and frequency trials, we are only 98% likely to recover more than 77% of a signal’s available power. Thus, in our worst case, we are 98% likely to recover at least \( (0.77)(0.81) P_{\text{sig}} = 0.62 P_{\text{sig}} \).

We account for this reduction in recovered signal power approximately by solving Equation 3.18 in the form

\[ p_n(P > P_{\text{det}}; 0.62P_{\text{sig}}) = 0.97. \]  

(3.19)

The probability of our recovered signal power exceeding 0.62\( P_{\text{sig}} \) and our total spectral power \( P \) exceeding \( P_{\text{det}} \) is then approximately \( (0.98)(0.97) \approx 0.95 \), giving our desired 95% confidence.

Solving Equation 3.19 (numerically) results in \( P_{\text{sig}} = 93.6 \). For a given observa-
tion, the minimum required pulsed fraction is then given by \( F = \left( 4N_i P_{\text{pul}} / N_s^2 \right)^{1/2} \) and converted to RMS by dividing by \( 2\sqrt{2} \). Thus, our 95% confidence detection sensitivities for observations 8 a, 10 a, 10 b, 11 a, 11 b, 11 c, 12 a, and 12 b are given by 0.60%, 1.2%, 1.2%, 2.7%, 2.9%, 3.6%, 5.6%, and 10%, respectively. We verified these numbers with Monte Carlo simulations.

Our search included spin frequencies up to 1024 Hz. Had we restricted our search to a smaller range of frequencies, corresponding to the previously reported pulsations, our detection sensitivities would not have been drastically different. To achieve the same detection significance, with far fewer trials, we require only a slightly reduced spectral power. And since the pulsed fraction limit depends essentially on the square root of the required power, the detection sensitivities are not terribly sensitive to the number of frequency trials in the search. Given Fox's best estimate for the pulsation frequency \( 549.76^{+0.05}_{-0.03} \) Hz (D. Fox 1999, private communication), if we had confined our search to a 5\( \sigma \) range (about 549.76 Hz and 274.88 Hz), our detection sensitivity in observation 10 a would have improved to 1.0%, while in observation 12 b the limit would have been 8.8%.

### 3.2.5 Verification of Procedures

Simulated data were used to test our search codes and to verify the sensitivity of the search. As an additional check, we applied our acceleration method to SAX J1808.4-3658 — the accreting millisecond pulsar (Wijnands & van der Klis, 1998; Chakrabarty & Morgan, 1998). Since our search was for a similar pulsar, with \( \sim \) few hundred Hz frequency in a \( \sim \) few hour orbit about a low-mass companion, SAX J1808.4-3658 provides a useful test of our acceleration method.

We analyzed an RXTE/PCA observation (observation #30411-01-01-04). We selected energy PHA bins corresponding to 0.4-17 keV from data of the same type we use in our search (E125). We barycenter corrected the arrival times with the position given by Roche et al. (1998), and used 2048 seconds of data beginning at Mission Elapsed Time (MET, seconds since 1994.0) 135395082.
The pulsations in these data are easily detected — the $\sim 401$ Hz signal is obvious in an unaccelerated PDS, with significant power detected in each of about 30 adjacent frequency bins, using 2048 seconds of data (Figure 3.1, dotted line). The highest single bin power represents about 22% of the total signal power in the extended feature.

Using the known orbital parameters (Chakrabarty & Morgan, 1998) to remove the acceleration from the data, the spreading of the signal power is reduced to about 6 independent Fourier bins around the correct rest frequency of the pulsar, 400.9752106(8) Hz, with $\sim 85\%$ of the total power in a single spectral bin (Figure 3.1, solid line). Oversampling reveals that the spreading that remains is consistent with the expected sinc-function response of the (discrete) FFT to a signal with the pulsar’s rest frequency (van der Klis, 1989). Thus, we confirm that our technique successfully removes the effect of the doppler shift due to the pulsar’s orbital motion.

### 3.2.6 Search Results

No candidate pulsations from any of the Aql X-1 observations exceeded our predetermined detection threshold. Using our largest detected search power, we can calculate upper limits on the strength of any pulsar signal contained in the various data sets searched (Vaughan et al., 1994), provided its orbit and spin frequency are covered by our search phase space. These upper limits are more restrictive than the a priori detection sensitivities.

The upper limit calculation is essentially the same as the detection sensitivity calculation, with the detection threshold power $P_{\text{det}}$ replaced by the maximum observed power $P_{\text{max}} = 30.96$. The results are shown in Table 3.3. For example, with 95% confidence, we can say that there is no sinusoidal pulsar signal in observation 10 a with a fractional RMS amplitude greater than 1.0%. The limits we can place on the fractional RMS become less stringent as the source flux decreases, increasing to an upper limit of 9.0% during observation 12 b, when Aql X-1 was observed to be faintest ($\sim 1$ mCrab; 13 000 PCA countss$^{-1}$=1 Crab).
Figure 3.1: SAX J1808.4-3658. The dotted line shows a section of the unaccelerated power spectrum. The spreading of the signal power due to the orbital doppler shift is evident. The solid line shows the power spectrum after removal of the doppler shift using our acceleration technique. The signal power has not been confined to a single bin because the pulsar frequency lies between two discrete Fourier frequencies. During this observation, SAX J1808.4-3658 passes through longitude = 0. The peak in the uncorrected spectrum is therefore maximally offset from the pulsar’s rest frequency, \( \Delta f_{\text{max}} = \beta f_0 = 0.022 \text{ Hz.} \)
Table 3.3: Aql X-1 observation parameters and search results

<table>
<thead>
<tr>
<th>Observation</th>
<th>MET Analyzed</th>
<th>Source Count Rate(^a)</th>
<th>Background Count Rate(^a,b)</th>
<th>Fractional RMS Amplitude(^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 a</td>
<td>99869572-98871620</td>
<td>670.6±0.8</td>
<td>37.3</td>
<td>&lt;0.52%</td>
</tr>
<tr>
<td>10 a</td>
<td>100220227-10022275</td>
<td>188.2±0.6</td>
<td>35.8</td>
<td>&lt;1.0%</td>
</tr>
<tr>
<td>10 b</td>
<td>100226227-100228275</td>
<td>177.4±0.6</td>
<td>35.8</td>
<td>&lt;1.1%</td>
</tr>
<tr>
<td>11 a</td>
<td>100474022-100476070</td>
<td>52.6±0.5</td>
<td>35.7</td>
<td>&lt;2.3%</td>
</tr>
<tr>
<td>11 b</td>
<td>100479522-100481570</td>
<td>48.6±0.5</td>
<td>35.7</td>
<td>&lt;2.5%</td>
</tr>
<tr>
<td>11 c</td>
<td>100485082-100487130</td>
<td>45.5±0.5</td>
<td>35.7</td>
<td>&lt;2.6%</td>
</tr>
<tr>
<td>12 a</td>
<td>100646431-100648479</td>
<td>19.0±0.5</td>
<td>31.2</td>
<td>&lt;4.9%</td>
</tr>
<tr>
<td>12 b</td>
<td>100652431-100654479</td>
<td>9.2±0.5</td>
<td>31.2</td>
<td>&lt;9.0%</td>
</tr>
</tbody>
</table>

\(^a\) counts s\(^-1\) (13 000 counts s\(^-1\) = 1 Crab)
\(^b\) Systematic uncertainty ~ 0.5 counts s\(^-1\)
\(^c\) Upper limits are 95% confidence

3.3 Discussion

We find no evidence of pulsations from Aql X-1 as it fades into quiescence. The upper limits on the pulsed fraction for the eight data sets searched range from 0.52% to 9.0% RMS. If, indeed, during the period identified as the “propeller phase” (Campana et al., 1998; Zhang et al., 1998a) the NS magnetic field significantly modifies the accretion flow geometry in the vicinity of the NS, the apparent absence of pulsations does not support the hypothesis that the quiescent emission is due to continued accretion. Our results cannot completely rule out such accretion, however, since it is possible that the geometry of the system may not lead to detectable pulsations; for example, the magnetic axis may be very nearly aligned with the rotation axis, or the rotation axis may point directly towards the earth. The non-detection of pulsations coupled with the observation of a thermal spectrum during quiescence (Rutledge et al., 1999) favors the interpretation that the quiescent luminosity is not due to accretion, but rather to a hot NS core (Brown et al., 1998).

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