Physics 106a: Classical Mechanics

Homework 2: Lagrangian Mechanics

Due: Thursday, October 14, 1999

Recommended reading: Goldstein pp. 20 – 21, 35 – 45
Supplemental reading: Feynman Lectures Vol. 2 19.1 – 19.9

1. *(Calculus of variations.)* Assume that you are on the surface of a cone with a half angle α which is a surface of revolution about the z-axis. Find an equation in plane polar coordinates for the geodesic curves (curve that is the shortest distance between two points) on this surface. Roll up a piece of paper into a cone and visualize these curves geometrically.

2. *(Degrees of freedom.)* A flexible chain of M massive point particles has rigid weightless rods as $M-1$ links. Each joint is free to move in any direction. How many degrees of freedom does the chain have? If you place the chain on a flat table, how many degrees of freedom does it have? Suppose the chain is lifted off the table and is closed by one more link. How many degrees of freedom does it have?

3. *(Kinetic energy.)* Consider the spherical pendulum, consisting of a mass m suspended by a string from the ceiling. The mass is free to swing in both directions but maintains a constant distance from the point of suspension. Choose spherical polar coordinates θ, ϕ as generalized coordinates for this problem. What is the kinetic energy, $T(\theta, \dot{\theta}, \phi, \dot{\phi})$?

4. *(Lagrangian, equations of motion.)* Consider a cylinder rolling down a plane inclined to the horizontal by an angle α. Let the radius of the cylinder be a and its radius of gyration be k. Write down the Lagrangian for the system and solve for the motion assuming that the cylinder does not slip on the plane. Evaluate the normal reaction force and show that the coefficient of friction μ must satisfy

$$\mu > \frac{k^2 tan(\alpha)}{k^2 + a^2} \qquad (1)$$

if the no-slip condition is to apply.

5. *(Potentials with scaling properties.)* Let $V(\vec{r_1}, ..., \vec{r_m})$ be the potential energy of a system of M massive particles with the scaling property

$$V(\alpha\vec{r_1}, ..., \alpha\vec{r_m}) = \alpha^k V(\vec{r_1}, ..., \vec{r_m}) \qquad (2)$$

(k is usually an integer, and α an arbitrary constant). Prove that, if the Lagrangian is to remain invariant (except for multiplication by a constant), and all distances are scaled by a factor α, then the time must be scaled by a factor $\beta = \alpha^{1-k/2}$. Applications of this include:

(a) if $k = 1$, the force is constant, like gravity. Prove that distances scale like t^2.

(b) if $k = 2$, the force is like that of a harmonic oscillator. Prove that the frequency is independent of the amplitude of oscillation.

(c) if $k = -1$, we have the Kepler problem (inverse square force law). Prove Kepler's third law from this scaling law above – that $d^3 = t^2$, where d could be any distance in the problem. Normally it is the mean distance of a planet from the Sun.

Extra Credit. Goldstein 2.4