Physics 106a: Classical Mechanics

Homework 3: Applications of Lagrangian Dynamics

Due: Thursday, October 21, 1999

Recommended reading: Goldstein pp. 45 – 63
Supplemental reading: Landau and Lifshitz 25 – 27, 80 – 83

1.*(Pendulum in an accelerated reference frame.)* A pendulum with a weightless string of length D and mass m is attached to a moving car. The car is continuously accelerated along a horizontal track with constant acceleration a, starting from an initial horizontal velocity v_o. Gravity acts in the vertical direction.

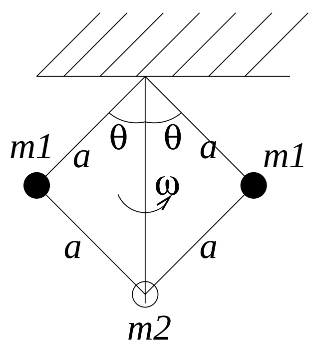
Assume that the (x,y) co-ordinate system is at rest with respect to the ground. Use the angle between the vertical and the pendulum bob, θ, as the generalized co-ordinate.

a.) Find the components of the velocity of the pendulum bob in the laboratory frame (the frame at rest with respect to the moving car). Find the kinetic energy as a function of $\theta, \dot{\theta}$, and the other variables (all of which are known functions).

b.) Find the Lagrangian $L(\theta, \dot{\theta}, t)$. Does L depend explicitly on time?

c.) Find the equation of motion for the pendulum.

2. *(Lagrangian, Energy Integrals.)* A flyball governer (for regulating motor speed) is comprised of two balls of mass m_1 attached by light struts of length a to a fixed bearing A, and by another pair of struts, also of length a to a mass m_2 that can slide freely on a vertical axle. The governer rotates about this axle with fixed angular velocity ω.



a.) Choosing the angle θ that a strut makes with the vertical as a coordinate, write down a Lagrangian.

b.) Use your Lagrangian to determine how θ varies with ω in an equilibrium solution.

c.) Find an energy integral.

d.) Find the frequency of small oscillations about the equilibrium.

3. *(Period of a plane pendulum.)* Show that for small m

$$K(m) = \pi/2(1 + m/4 + 9m^2/64 + ...) \tag{1}$$

Thus, if a pendulum clock beats seconds when its maximum swing is 10°, how many minutes per day will it lose if its amplitude is increased to 20°?

Extra Credit. Consider the full non-linear equations that we derived in class for the double pendulum. Write a computer program to solve them numerically, and explore solutions for large amplitude motion. Are they always periodic?