Physics 106a: Classical Mechanics

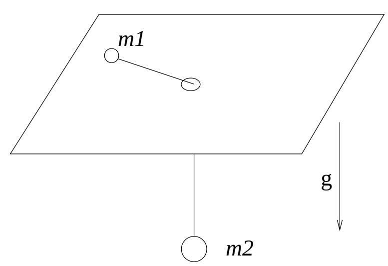
Homework 4: Central Force Motion

Due: Thursday, October 28, 1999

Recommended reading: Goldstein pp. 70 – 105

1.(*Center of mass motion.)* Two massive bodies move in a constant external gravitational field. Show that their motion can be reduced to an equivalent one-body problem.

2.(*Effective potential.)* Two point masses, m_1 and m_2 are connected by a string. One of the masses, m_1, moves on the (horizontal) surface of a table, while the other mass, m_2, hangs suspended by the string below the table. The string passes through a hole in the center of the table (see sketch). Assume m_2 moves only in the vertical direction, and neither m_1 nor m_2 pass through the hole.



a.) The second order equation derived for this system in class can be interpreted as the equation of motion for a single particle of mass m moving in 1-D in an effective potential $U(x)$. Find $m, U(x)$, and make a qualitative sketch of $U(x)$. What physical condition must be satisfied for m_1 to move in a circle? What is the radius of the circle?

b.) Obtain a first-order equation for the motion.

3. *(Motion in a gravitational field.)* Two stars are in a circular orbit about a common, stationary center of mass. One of them has a supernova explosion resulting in it losing a mass ΔM spherically symmetrically in its frame in a time short compared with the orbital period and leaving behind a system with combined mass M. Show that the relative orbit is bound and develops an eccentricity $e = \Delta M/M$ provided that less than half the combined mass is lost. Show, also, that if the orbit is unbound, the supernova remnant star moves with constant speed.

4. *(Hyperbolic kick.)*

a.) Why does an object in a hyperbolic orbit passing close to a planet (which is in orbit about another large object like the Sun) get a velocity "kick" (called a gravity assist) from it?

b.) Why does it not work for a stationary planet?

5. (*Cross section.)* A point particle off mass m moving with velocity v_1 leaves a half-space in which its potential is a constant U_1 and enters another in which its potential energy is a different constant U_2.

a.)Show that

$$\sin\theta_1/\sin\theta_2 = \sqrt{1 + 2(U_1 - U_2)/(mv_1^2)} \tag{1}$$

where θ_1 and θ_2 are the angles between the normal to the plane and the initial and final velocities.

b.) Now consider a stationary spherical “potential well” of radius a and “depth” U_o (i.e. a field with $U = 0$ for $r > a$ and $U = -U_o$ for $r > a$). Use the result from above to find a relationship between the scattering angle Φ and impact parameter b (for both $b > a$ and $b < a$) for an incident point particle of mass m and velocity at infinity v_o.

c.) Find the differential cross section for scattering.

d.) What is the total effective cross section?

(Extra Credit). Write a computer program to find the orbit (by numerical integration) for an attractive potential of the form $1/r^n$. Verify Bertrand’s theorem “experimentally” on the computer for yourself.