Physics 106a: Classical Mechanics

Homework 7: Rigid Body Dynamics

Due: Thursday, December 2, 1999 Note, you have two weeks to complete this homework. There will be no extensions granted, except for compelling circumstances, since we must have the time to grade this before the final.

Recommended reading: Goldstein pp. 188 - 232 (Eq. 5–88 and 5–89 are wrong).

1. (*Orthogonal transformation represented as rotation.)* Prove that any orthogonal transformation **U** can be represented as a rotation through an angle about an axis that is left unchanged by **U**.

2. (*I for an ellipsoid.)* Find the inertia tensor, principal axes, and principal moments for an ellipsoid of semiaxes a, b, c.

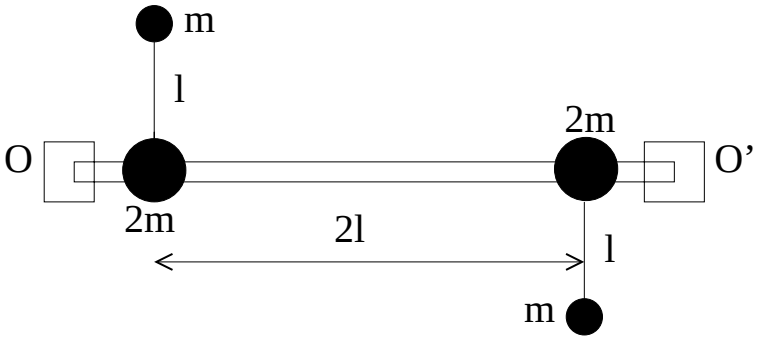
3. *(Yet more rolling cones.)* Goldstein 5.6

4. *(Equations of motion for a rotating rod.)* Goldstein 5.13

5. *(Constraints for a rolling sphere.)* Goldstein 5.17

6. *(Torque-free symmetric top.)* Goldstein 5.19 a) and b).

7. *(Cam shaft.)* The figure shows a simple-minded abstraction of a cam shaft, with point masses m and $2m$ fixed on massless rods, all in a plane. The shaft rotates with constant angular velocity, ω, around the axis O-O′ through the long shaft, held by frictionless bearings at O and O′.



a) Calculate all elements of the inertia tensor. Be sure to specify the coordinate system you use.
b) Using the elements just calculated, find the angular momentum of the cam shaft in the lab frame as a function of time.
c) What is the torque with respect to the mid-point of the long shaft, exerted by the bearings? Give magnitude and direction in the lab frame.
d) Locate an axis, fixed in the plane of the masses around which the cam shaft could rotate with zero torque when the angular velocity is constant.