

Physics 106C: Electromagnetism

Homework 2: Multipole Expansions

DUE: Thursday, April 19 2001

Remember: Late homework will be granted 50% credit UNLESS PRIOR ARRANGEMENTS ARE MADE WITH ME OR A TA. If you have an extension, please indicate who granted it clearly on the top of the paper OR YOU WILL NOT GET FULL CREDIT. Note that for the rest of Phys106, you must solve integrals analytically, or by looking them up in a table, but you may not use Mathematica.

Reading Assignment: Jackson Chapter 4.1 - 4.2

1) Show that the average field inside a sphere of radius R due to all the charge within the sphere is

$$\mathbf{E}_{ave} = -\frac{1}{4\pi\epsilon_0} \frac{\mathbf{p}}{R^3}$$

2) Calculate the first two non-vanishing multipole moments inside a sphere of diameter d and outside a sphere of diameter $\sqrt{(2a)^2 + d^2}$ for the charge density

$$\begin{aligned} \rho(r) &= \frac{Q}{\pi a^2} \left(\delta\left(z - \frac{d}{2}\right) - \delta\left(z + \frac{d}{2}\right) \right), \rho < a \\ &= 0, \rho > a \end{aligned}$$

where $\rho = \sqrt{x^2 + y^2}$

3) Derive the exact potential for a spherical shell of radius R carrying a surface charge $\sigma = k \cos \theta$

a) calculate the dipole moment of this charge distribution.

b) Find the approximate potential at points far from the sphere, and compare to the exact answer. What can you conclude about the higher multipoles?

Do Jackson problems 4.1 and 4.4