

Physics 106C: Electromagnetism

Homework 6: Faraday's Law, Time-Variable Fields

DUE: Thursday, May 24 2001

Remember: Late homework will be granted 50% credit UNLESS PRIOR ARRANGEMENTS ARE MADE WITH ME OR A TA. If you have an extension, please indicate who granted it clearly on the top of the paper OR YOU WILL NOT GET FULL CREDIT. Note that for the rest of Phys106, you must solve integrals analytically, or by looking them up in a table, but you may not use Mathematica.

Reading Assignment: Jackson 5.15 – 5.18, 6.1 – 6.3, 6.6 – 6.7

1. Use the stress tensor to calculate the force per unit length between two very long, straight, parallel thin wires separated by a distance d and carrying currents I_1 and I_2 respectively in a large space filled with material of permeability μ . Note that the current may be in the same or opposite directions. (Choose the surface over which you calculate and integrate the stress tensor to make the calculation as simple as possible).

2. Imagine that an iron sphere of radius R carrying a charge Q and a uniform magnetization $\mathbf{M} = M\hat{k}$. The sphere is initially at rest.

a) Compute the angular momentum stored in the electromagnetic fields.

b) Suppose the sphere is gradually (and uniformly) demagnetized. Use Faraday's law to determine the induced electric field, find the torque this field exerts on the sphere, and calculate the total angular momentum imparted to the sphere in the course of the demagnetization.

c) Suppose instead of demagnetizing the sphere we discharge it, by connecting a grounding wire to the north pole. Assume the current flows over the surface in such a way that the charge density remains uniform. Use the Lorentz force law to determine the torque on the sphere, and calculate the total angular momentum imparted to the sphere in the course of the discharge. (the magnetic field is discontinuous at the surface, does this matter?)

3. It is always possible to pick a vector potential whose divergence is zero (Coulomb gauge). Show that it is always possible to choose

$$\nabla \cdot \mathbf{A} = -\mu_o \epsilon_o (\partial \Phi / \partial t), \quad (1)$$

as required for the Lorentz gauge, assuming you know how to solve equations of the form

$$\nabla^2 \mathbf{A} - \mu_o \epsilon_o \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_o \mathbf{J} \quad (2)$$

$$\nabla^2 \Phi - \mu_o \epsilon_o \frac{\partial^2 \Phi}{\partial t^2} = -\frac{1}{\epsilon_o} \rho \quad (3)$$

Is it always possible to pick $\Phi = 0$? How about $\mathbf{A} = 0$?

4. A current I flows through a cylindrical resistor with resistance R . Show that the surface integral of the inward component of the Poynting vector yields the Joule heating RI^2 .