

Physics 106A: Classical Mechanics

Homework 1: Newton's Laws and Calculus of Variations

DUE: Thursday, October 10 2002

Remember: Late homework will be granted 50% credit up to 1 week late. After that, no credit will be given.

Reading Assignment: Hand and Finch Chapter 2.

1. (*Newton's laws.*) A small block of mass m slides on a frictionless inclined plane of mass M . The plane has height h , makes an angle α with the horizontal, and rests on a flat, frictionless surface. The small block can slide down the plane, and the plane can slide along the surface. If the small block starts at the top of the plane, how long will it take to reach the bottom? (You will no doubt notice the solution to this problem in your textbook. Attempt to solve it yourself using Newton's laws - refer to the solution if you get stuck.

2. (*Newton's laws.*) Show that for a single particle with constant mass the equation of motion implies the following differential equation for the kinetic energy:

$$\frac{dT}{dt} = \vec{F} \cdot \vec{v} \quad (1)$$

while if the mass varies with time the corresponding equation is

$$\frac{d(mT)}{dt} = \vec{F} \cdot \vec{p} \quad (2)$$

3. (*Newton's laws.*) Prove that a bead that is placed anywhere on a vertical frictionless wire in the form of a cycloid satisfying

$$x = h(\theta + \sin \theta), \quad z = h(1 - \cos \theta); \quad -\pi < \theta < \pi \quad (3)$$

(where x is a coordinate advancing in the horizontal direction and z advances in the vertical direction), will reach the bottom in the same time regardless of the starting point and find that time.

4. (*Calculus of variations.*) Hand and Finch Chapter 2, Problem 2.