Physics 106A: Classical Mechanics
Homework 2: Lagrangian Formulation of Mechanics

DUE: Thursday, October 17 2002

Remember: Late homework will be granted 50% credit up to 1 week late. After that, no credit will be given.

Reading Assignment: Hand and Finch Chapter 1, 2.

1. (*Degrees of Freedom.*) Hand and Finch chap. 1, prob. 2

2.(*Block on an Inclined Plane.*) Take the block and inclined plane problem from homework 1.
a) Write down the Lagrangian for the system (block and plane).
b) Apply the Euler-Lagrange equations to find the equations of motion for the system.
c) From the equations of motion, find the time it takes the block to slide down the plane.

3. (*Point transformation*) Let $q_1, ..., q_n$ be a set of independent generalized coordinates for a system of n degrees of freedom, with a Lagrangian $L(\vec{q}, \dot{\vec{q}}, t)$. Suppose we transform to another set of independent coordinates $s_1, ..., s_n$ by means of transformation equations

$$q_i = q_i(s_1, ..., s_n, t), \quad i = 1, ..., n. \tag{1}$$

A transformation like this is called a point transformation. Show that if the Lagrangian function is expressed as a function of $s_j, \dot{s_j}$, and t through the equations of transformation, then L satisfies Lagrange's equations with respect to the s coordinates:

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{s}_j}) - \frac{\partial L}{\partial s_j} = 0 \tag{2}$$

In other words, the form of Lagrange's equations is invariant under a point transformation.

4. (*Virtual work*) Hand and Finch Chapter 1 Problem 5