Physics 106A: Classical Mechanics
Homework 6: Small Oscillations and Normal Modes

DUE: Thursday, November 21 2002

Remember: Late homework will be granted 50% credit up to 1 week late. After that, no credit will be given.

Reading Assignment: Hand and Finch Chapter 9.

1. (*Diagonalizing L.*) Using the modified eigenvalue equation,

$$\mathbf{V} \cdot \vec{a}_k = \lambda_k \mathbf{T} \cdot \vec{a}_k$$

show that the transformation into normal mode eigenvectors diagonalizes both $\mathbf{T}$ and $\mathbf{V}$. Show futher that if $\Theta_{\mathbf{k}}$ are the normal co-ordinates, defined by

$$\vec{\theta_k} = Re[c_k \vec{a_k} e^{i\omega_k t}]$$

we can write the Lagrangian as

$$L = \frac{1}{2} m_k (\dot{\vec{\Theta}}_k^{\,2} - \omega_k^2 \vec{\Theta}_k^{\,2})$$

Note: in the first expression defining the normal co-ordinates, k is not summed over, but in the expression for the Lagrangian it is.

2. (*Double Pendulum Again)*) Obtain the normal modes of vibration for the double pendulum assuming equal lengths, but not equal masses. Show that when the lower mass is small compared to the upper one the two resonant frequencies are almost equal. If the pendula are set in motion by pulling the upper mass slightly away from the vertical and then releasing it, show that subsequent motion is such that at regular intervals one pendulum is at rest while the other has its maximum amplitude (just like the beat phenomenon we discussed in class).

3. (*Two Masses, Three Springs*) Two masses are connected by three springs as shown in the figure. The equilibrium length of the sprincs is a. The masses are free to move in the horizontal direction only. Find the eigenfrequencies and normal modes of the system. You are free to use the symmetry of the problem to guess the normal modes, as long as you verify your guess.