

# Physics 106A: Classical Mechanics

## Homework 7: Rotating Co-ordinate Systems, Rigid Body Dynamics

**DUE: Friday, December 6 2002, 5:00 p.m. in 203 Downs**

NOTE: You have two weeks to complete this assignment. It is long, so don't leave it until the last week! Note the unconventional due date. I will hold an additional help session for this HW after class Thursday, 12/5 from noon - 1:00 p.m..

Reading Assignment: Hand and Finch Chapter 7,8

Problem 1: (*Rolling Sphere*) Hand and Finch Chapter 7, problem 4. Are the constraints holonomic?

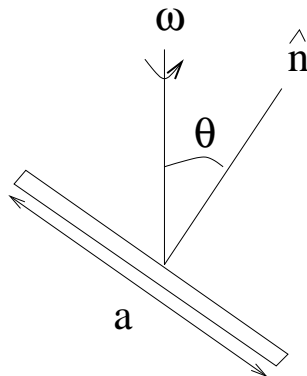
Problem 2: (*Orthogonal Matrices*) Hand and Finch Chapter 7, problem 7

Problem 3: (*Coriolis Force*) Hand and Finch Chapter 7, problem 11

Problem 4: (*Moment of Inertia*) Hand and Finch Chapter 8, problem 7

Problem 5: (*Rolling Cones*) A uniform right circular cone of height  $h$ , half-angle  $\alpha$ , and density  $\rho$  rolls on its side without slipping on a uniform horizontal plane in such a manner that it returns to its original position in a time  $\tau$ . Find expressions for the kinetic energy and the components of the angular momentum of the cone.

Problem 6: (*Rotating Plate*) A thin, uniform, square plate with side length  $a$  rotates at constant angular frequency  $\omega$  about an axis through the center, but tilted at an angle  $\theta$  with respect to the normal to the plate (see sketch).



- Find the principal moments of inertia, and sketch the principle axes.
- Find an expression for the angular momentum,  $\vec{L}$ , in the laboratory frame.
- Calculate the torque on the axis, expressed in the laboratory co-ordinate system.

Problem 7: (*Symmetric Top*) Show that the angular momentum of the torque-free symmetrical top rotates in the body coordinates about the symmetry axis with an angular frequency  $\Omega$ . Show also that the symmetry axis rotates in space about the fixed direction of the angular momentum with the angular frequency

$$\dot{\phi} = \frac{I_3 \omega_3}{I_1 \cos \theta}$$

where  $\phi$  is the Euler angle of the line of nodes with respect to the angular momentum as the space  $z$  axis.