Physics 106A: Classical Mechanics
Homework 6: Small Oscillations/Normal Modes

DUE: Thursday, November 16 2000

Remember: Late homework will be granted 50% credit UNLESS PRIOR ARRANGEMENTS ARE MADE WITH
ME OR A TA. If you have an extension, please indicate who granted it clearly on the top of the paper.

Reading Assignment: Hand and Finch Chapter 9

1. (Double Pendulum Again) Obtain the normal modes of vibration for the double pendulum assuming equal lengths, but not equal masses. Show that when the lower mass is small compared to the upper one the two resonant frequencies are almost equal. If the pendula are set in motion by pulling the upper mass slightly away from the vertical and then releasing it, show that subsequent motion is such that at regular intervals one pendulum is at rest while the other has its maximum amplitude (like the beat phenomenon we discussed in class).

2. (Pendulum attached to a block) Hand and Finch Chapter 9 problem 1

3. (Two masses coupled by springs.) Hand and Finch Chapter 9 problem 3

4. (5-atom molecule) A 5-atom linear molecule is simulated by 5 masses connected by four springs, as shown below (note all the masses are not equal). All force constants are the same. Find the eigenfrequencies and normal modes for longitudinal vibration. Hint: transform the coordinates $\eta_i$ for the displacements to $\xi_i$ defined by

$$
\eta_3 = \xi_3 \quad \eta_1 = \frac{\xi_1 + \xi_5}{\sqrt{2}} \quad \eta_5 = \frac{\xi_1 - \xi_5}{\sqrt{2}}
$$

with symmetrical expressions for $\eta_2$ and $\eta_4$. The secular determinant will then factor into determinants of lower rank.

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