

Physics 106A: Classical Mechanics

Homework 7: Rotating Co-ordinate Systems, Rigid Body Dynamics

DUE: Friday, December 1 2000, 5:00 p.m.

NOTE: You have two weeks to complete this assignment. It is long, so I do not recommend leaving it until the last week! THERE WILL BE NO EXTENSIONS ON THIS ASSIGNMENT (except in the case of a medical excuse/emergency).

Reading Assignment: Hand and Finch Chapter 7,8

Problem 1: (*Angular Velocity*) Hand and Finch Chapter 7 problem 1

Problem 2: (*Rolling Sphere*) Hand and Finch Chapter 7, problem 4

Problem 3: (*Orthogonal Matrices*) Hand and Finch Chapter 7, problem 7

Problem 4: (*Coriolis Force*) Hand and Finch Chapter 7, problem 12

Problem 5: (*Moment of Inertia*) Hand and Finch Chapter 8, problem 7

Problem 6: (*Rolling Cones*) A uniform right circular cone of height h , half-angle α , and density ρ rolls on its side without slipping on a uniform horizontal plane in such a manner that it returns to its original position in a time τ . Find expressions for the kinetic energy and the components of the angular momentum of the cone.

Problem 7: (*Equations of motion*) A uniform bar of mass M and length $2l$ is suspended from one end by a spring of force constant k . The bar can swing freely only in one vertical plane, and the spring is constrained to move only in the vertical direction. Set up the equations of motion in the Lagrangian formulation.

Problem 8: (*Symmetric Top*) Show that the angular momentum of the torque-free symmetrical top rotates in the body coordinates about the symmetry axis with an angular frequency Ω . Show also that the symmetry axis rotates in space about the fixed direction of the angular momentum with the angular frequency

$$\dot{\phi} = \frac{I_3 \omega_3}{I_1 \cos \theta}$$

where ϕ is the Euler angle of the line of nodes with respect to the angular momentum as the space z axis.