

Physics 106A - Classical Mechanics
Presentation Problem #1
For Tues., Oct. 10 2000

Prepare the answer to this problem for presentation to the class. It is your goal not only to solve the problem yourselves, but to explain the solution clearly, in such a way that your colleagues will understand. Be prepared to discuss your reasoning.

Deducing the Lagrangian for a free particle:

In deriving the Lagrange equations from Hamilton's principle, we introduced the function L . If we know nothing about Newton's laws, we must *deduce* the form of L . Let's do this for a particle in free space. We must use what we know about inertial systems - that free space is homogeneous and isotropic.

A) Explain why, on very general grounds, L cannot be a function of x, y , or z . Also, explain why it can't depend on the individual coordinates of velocity in any way except as on the magnitude ($v^2 = v_x^2 + v_y^2 + v_z^2$).

B) Adopt the postulate that all inertial frames are equivalent, and also adopt the Euler-Lagrange equations as derived from Hamilton's principle as an accurate description of nature. Assume an inertial frame K . A second inertial frame K' moves at constant velocity $-V_0$ with respect to K , so that the transformation law of velocities is

$$\vec{v}' = \vec{v} + \vec{V}_0$$

Explain why these postulates mean that

$$L'(v + V_0) = L(v) + \frac{dF(x, t)}{dt}$$

$L(v)$ is an unknown function for the free particle that we are trying to determine from these principles. Working in one dimension to make things easier. Let V_0 be an infinitesimal quantity. Expand the left side of the above equation in a Taylor series and keep only the first two terms. From this prove $L(v) \sim v^2$. Since we can choose any multiplicative constant, $L = T$ is a valid Lagrangian. Given this form for the Lagrangian, what do the Euler-Lagrange equations imply about the motion of a particle in free space?

C) Now consider the effect of a conservative force, which is derivable from the gradient of a scalar potential $V(\vec{r}, t)$. This potential is velocity-independent. This force can cause the velocity to change, so L can now depend on \vec{r} . Assume this adds a term to L which depends on \vec{r} . Show that the Euler-Lagrange equations together with Newtonian mechanics imply

$$L(\vec{r}, \vec{v}, t) = T - V$$