

Physics 106A - Classical Mechanics
Presentation Problems #7,8,9,10
For Sat., Dec. 2 2000 – 1:00 p.m.

Assigned to group 7,8,9,10 (in that order)

Prepare the answer to this problem for presentation to the class. It is your goal not only to solve the problem yourselves, but to explain the solution clearly, in such a way that your colleagues will understand. Be prepared to discuss your reasoning.

Problem 7: A canal in the northern hemisphere has width W and carries water in a due northward direction at speed v .

(a) Sketch a qualitative diagram of the forces acting on a fluid element (neglecting viscosity), identifying them by name (e.g. gravitational force, etc.).

(b) For latitude λ (the equator has $\lambda = 0$), calculate how much higher the water comes up on one bank than the other. Your solution must show which bank (east or west) has the higher water line. Calculate approximately what the height difference is for $W = 20\text{m}$ and $v = 5\text{m/s}$ at latitude 45° (use $\omega_{\text{earth}} = 7.3 \times 10^{-5}$ rad/s). *Hint: the surface of the water must be an isobar (at constant pressure).*

(c) From the point of view of an observer at rest in an inertial (non-rotating) frame of reference, what would be the cause of the observed tilt in the water level?

Problem 8: Hand and Finch Chapter 8 problem 20.

Problem 9: Klingon engineers have constructed a fiendish trap for unsuspecting, passing spaceships. It consists of a “fifth force”, spherical attractive well that can be approximated by the potential

$$V(r) = \frac{-Km}{r^4}; K > 0 \quad (1)$$

where m is the mass of the spaceship (assumed to be much smaller than that of the trap). The Starship Enterprise approaches the trap with speed v_∞ and shuts down its engines in order to avoid detection.

(a) What is Captain Kirk’s minimum prudent impact parameter in order to avoid capture?

(b) The trap can also act as a cosmic “vacuum cleaner” sweeping up interstellar dust. If the dust can be regarded as having uniform density ρ and negligible random motion, calculate the rate of collection of dust, expressed as a mass per unit time, as a function of the speed of the trap.

Problem 10: A particle of mass m moves under the influence of gravity but without friction on the surface of a cone of half angle α (see sketch).

(a) Write down the Lagrangian for the system in plane polar coordinates, and identify any conserved quantities.

(b) Find the equation of motion for the radial coordinate.

(c) The motion can be described by 1-D motion in an effective potential. Identify the effective potential, and sketch it as a function of the radial coordinate. Comment on the character of the motion (i.e. is it bounded, unbounded?).

(d) If the particle is launched with initial velocity v_o from height z_o , find the condition for circular motion.

