# 1 Topic 2: The Lagrangian Formulation of Mechanics

Reading Assignment: Hand & Finch Chap. 1 & Chap. 2

For the next week, we will be developing an alternate formulation of mechanics to Newton's laws, the *Lagrangian formulation.*

Before introducing Lagrangian mechanics, lets develop some mathematics we will need:

## 1.1 Some methods in the Calculus of Variations

In the calculus of variations we are studying "functions of functions", called *functionals.*

Euler considered a completely mathematical problem: we want to determine the function, $y(x)$ such that the integral

$$J = \int_{x_1}^{x_2} f\{y(x), y'(x) : x\} dx$$

is an extremum (min or max).

Note: $J$ is a function of two *functions* $y$ and $y' = dy/dx$ which are dependent variables, and the independent variable $x$.

$f$ is given, $y$ and $y'$ are varied to find an extremum (such that small variations in $y, y'$ produce only second-order changes in $J$ ). We can vary $y$ and $y'$ independently.

What do we mean by varying $y$ (which is a function)?

Think of a "neighbouring function", which we define with the following parametric representation:

$$\begin{aligned} y &= y(\alpha, x) \\ &= y(0, x) + \alpha\eta(x) \end{aligned}$$

$\eta(x)$ is a function with continuous first derivative. The endpoints, $x_1, x_2$ are fixed (or finding an extremum would be an undefined problem), and so we know that $\eta(x_1), \eta(x_2) = 0$.

A stationary value for the integral (ie condition for an extremum) requires

$$\frac{\partial J}{\partial \alpha}|_{\alpha=0} = 0$$

This is a necessary but not sufficient (there can be saddle points) condition.

### 1.1.1 Derivation of Euler's equations

Condition for an extremum:

$$\begin{aligned}
\frac{\partial J}{\partial \alpha} &= \frac{\partial}{\partial \alpha}\int_{x1}^{x2} f(y,y';x)dx \qquad\qquad y = y(\alpha, x) \\
&= \int_{x1}^{x2}\left(\frac{\partial f}{\partial y}\frac{\partial y}{\partial \alpha} + \frac{\partial f}{\partial y'}\frac{\partial y'}{\partial \alpha}\right)dx \\
(\ y &= y_0 + \alpha\eta) \\
&= \int_{x1}^{x2}\left(\eta\frac{\partial f}{\partial y} + \eta'\frac{\partial f}{\partial y'}\right)dx
\end{aligned}$$

integrate the second term by parts:

$$\int \eta'\frac{\partial f}{\partial y'}dx = \eta(x)\frac{\partial f}{\partial y'}|_{x1}^{x2} - \int_{x1}^{x2}\eta\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)dx$$

$$\frac{\partial J}{\partial \alpha} = 0 = \int_{x1}^{x2}\eta(x)\left(\frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'}\right)dx$$

since $\eta(x)$ is an arbitrary function, we must have

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0 \qquad \text{Euler's equation}$$

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Example: The brachistochrone problem (first solved by J. Bernoulli)

sketch: a particle moves in a constant force field from x1,y1 -> x2,y2

**if the particle starts at rest, what path allows transit in the minimum time?**

First, let's introduce a useful shorthand; $\delta =$ shorthand for making small variations in a function at constant value of the independent variable ($x$).

In this problem $t$ is a function of $y; x$ and we want to find an extremum of $t$ as we vary the path $y$:

$$\delta t = \int \delta f \ dx = 0$$

To find $f$ , use conservation of energy:

$$\begin{aligned}
T &= 1/2mv^2, U = -mgx,\ T + U = const. = 0 \\
&\rightarrow\ v = \sqrt{2gx}
\end{aligned}$$

and the time required to get from $x1, y1 \to x2, y2$ is

$$\begin{aligned} t &= \int \frac{ds}{v} = \int \frac{\left(dx^2+dy^2\right)^{1/2}}{\sqrt{2gx}} \\ &= \frac{1}{\sqrt{2g}} \int_0^{x2} \frac{\left(1+y'^2\right)^{1/2}}{\sqrt{x}} dx \end{aligned}$$

so, we identify $f = \sqrt{\left(1+y'^2\right)/x}$ and, using Euler's equation:

$$\begin{aligned} 0 &= -\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) \\ \frac{\partial f}{\partial y'} &= const \equiv \sqrt{2a} \end{aligned}$$

differentiate and square the result:

$$\frac{1}{2a} = \frac{y'^2}{x\left(1+y'^2\right)}$$

$$y = \int \frac{x dx}{\sqrt{2ax - x^2}}$$

in order to do the integral, make a change of variables: $x = a(1-\cos\theta), dx = a\sin\theta d\theta$

$$y = \int a\left(1-\cos\theta\right) d\theta = a(\theta - \sin\theta) + const.$$

this is the parametric equation for a cycloid passing through the origin

$$x = a(1-\cos\theta),\ y = a(\sin\theta)$$

sketch resulting curve – we adjust $a$ to make the curve go through $x2, y2$.

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Generalize Euler equations to several dependent variables

$$f = f\{y_i, y_i'; x\} \qquad\qquad i = 1, 2, ...n$$

where

$$y_i(\alpha, x) = y_{i0} + \alpha\eta_i\left(x\right)$$

the derivation goes exactly as before, with

$$\frac{\partial J}{\partial \alpha} = 0 = \int_{x1}^{x2} \eta_i(x) \left( \frac{\partial f}{\partial y_i} - \frac{d}{dx}\frac{\partial f}{\partial y_i'} \right) dx$$

$$\frac{\partial f}{\partial y_i} - \frac{d}{dx}\left(\frac{\partial f}{\partial y_i'}\right) = 0 \quad for \quad i = 1, 2....n$$

But, with one important condition: all the $\delta y's$ or $\eta_i's$ *must be independent* - so we can set the contents of the $()'\, s = 0$. If the $\eta_i's$ depend on one another, then we cannot in general solve the problem without introducing information about the interdependence of the $\eta_i's$. There are techniques to do this which we will learn later.

## 1.2 The Lagrangian Formulation of Mechanics

The Lagrangian formulation of mechanics is an alternative to the classical formalism, which is based on Newton's laws, but leads to the same equations of motion more quickly.

We apply Newton's laws in cartesian co-ordinates - we resolve orthogonal components of force - if the system is physically constrained, *the constraint forces come explicitly into the equations.*

– often we have constrained mechanical systems with fewer d.o.f. than the # of cartesian co0ordinates. We would like to reduce the # of variables.

– the Lagrangian formalism is based on energies rather than forces, so above is easy to do. Also, formalism is therefore independent of co-ordinate transformations → starting point for GR where cartesian co-ordinates don't exist. Also for QM (a description based on energies)

– is deeper physically and leads conceptually to QFT.

### 1.2.1 Degrees of Freedom and Constraints

The most general set of dynamical variables is any set that can completely describe the configuration of a mechanical system. Call these variables $q_k$ – these can be varied *independently* (unlike constrained cartesian co-ordinates).

→ # of variables = # d.o.f. of the system
the variables are usually angles or positions

*constraints* reduce the number of degrees of freedom
$j$ constraints, $M$ point particles
→ d.o.f $\quad N = 3M - j$

the spatial co-ordinates, $\vec{r}$, can be expressed as functions of the *generalized co-ordinates*

$$\vec{r}_i = \vec{r}_i(q_1, q_2, .....q_N, t)$$

note: $\vec{r} = r_1, ....r_k$ ; $k \geq N$

examples: pendulum – 1 d.o.f. $q = \theta$

bead on a wire – $q = x$

**look at types of constraints:**

*Holonomic constraints*:

we can have explicit time dependence of the constraints. For example:

pendulum, or bead on a wire:

$$\vec{r} = r(q_1, ....q_N) \qquad \text{sclerenomic}$$

if the wire or pivot moves

$$\vec{r} = \vec{r}(q_1, ....q_N, t) \qquad \text{rheonomic}$$

both these are *holonomic* in that they can be expressed as $f_\alpha(\vec{q}, t) = 0$. i.e. there is no dependence on velocities, $\dot{q}$.

*Non-holonomic constriants*

these constraints depend on velocities – (ball rolling without slipping on a table top), or involve inequalities (bead rolling off a sphere – $r \geq R$).

With this introduction, we will now derive the Lagrangian formulation two ways:

### 1.2.2 Deductive Approach – Hamilton's Principle

We start from a new axiom: *Dyamical systems follow a dynamical trajectory that extremizes their actions.*

Definition of action (a functional):

$$S = \int_{t_1}^{t_2} L\, dt$$

*L - Lagrangian* - a function of all the trajectories that could have been followed. A characteristic of the system.

*S - Action* - a functional. As we discussed in the calculus of variations, a function whose domain is a set of functions. These functions are the minimum set of co-ordinates needed to specify the state of the system - the *generalized coordinates* $q_1(t), q_2(t)$ ... we will refer to them as $\vec{q}(t)$.

Newtonican dynamics deals with second order differential equations, so we will need time derivatives as well. So, in general:

$$L = L(\vec{q}, \frac{d}{dt}\vec{q}, t)$$

We may regard the $\vec{q}'s$ as co-ordinates of a point in N-dimensional *configuration space* - the motion of the system traceds out a path in configuration space (i.e. time is a parameter).

A consequence of Hamilton's principle: extermize the action:

$$\delta \int_{t1}^{t2} L(\vec{q}, \frac{d}{dt}\vec{q}, t) dt = 0$$

This is exactly the kind of problem variational calculus tells us how to solve –

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = 0 \qquad \text{for i =1...N the } \textit{Euler-Lagrange Eqns}$$

*remember:* we must have all the $\delta q's$ be *independent*, to separate the equations and set them individually to = 0. So, # of $q's$ must equal the # d.o.f. of the system!

### 1.2.3 What is the Lagrangian?

The form of the Lagrangian is not set by Hamilton's principle. So far, there is little physics in what we have done – this comes when we deduce the form of $L$. This is a very general approach - in QFT we use all symmetries we know about to deduce the form. Next tuesday, your colleagues will show how to deduce the form of the Lagrangian for a free particle from the properties of free space.

### 1.2.4 Some properties of the Lagrangian:

1) The Euler-Lagrange equations are linear in $L$, so it can only be defined to a multiplicative constant.

2) Suppose

$$L \to L + \frac{dF(q,t)}{dt}$$

then,

$$S = \int_{t1}^{t2} L\,dt \to S + F\{\vec{q}(t_2), t_2\} - F\{\vec{q}(t_1), t_1\}$$

i.e. we have just added a constant to $S$. So, adding any $\frac{dF(q,t)}{dt}$won't change the dynamics (the E-L equations remain unchanged).

### 1.2.5 Deducing the form of the Lagrangian:

Now we must figure out what $L$ is. We leave it for a presentation problem to show that using the properties of free space (homogeneity and isotropy)

$$L = \frac{1}{2}mv^2 = T \qquad \text{for free particle in an inertial frame}$$

And, for a particle under the influence of a *conservative, velocity-independent* potential $V$:

$$L(\vec{r}, \vec{v}, t) = T - V \qquad \text{(note sign)}$$

(this will fortunately also be demonstrated by your colleagues on Tuesday).

## 1.3 A Second Approach to the Lagrange Formalism - Inductive

This approach will use Newton's laws in a more explicit way to derive the new formalism. Will use a construct called *virtual work* to derive the relevant partial differential equation.

### 1.3.1 Principle of Virtual Work for Statics

First, to introduce virtual work, consider the static case of a particle at rest under forces: the force on particle $i$ is $\vec{F}_i = 0$. Make an infinitesimal displacement, $\delta\vec{r}_i$. The condition for equilibrium gives:

$$\vec{F} \cdot \delta\vec{r}_i = 0$$

sum over all particles:

$$\sum_i \vec{F}_i \cdot \delta\vec{r}_i = 0$$

Now, separate into *external forces which can do work*, and *"constraint" forces which can't* (ie. we are considering only constraint forces of the sort which can't do work).

$$\sum_i \vec{F}_i^{ext} \cdot \delta\vec{r}_i + \vec{f}_i \cdot \delta\vec{r}_i = 0$$

$$\Longrightarrow \sum_i \vec{F}^{ext} \cdot \delta\vec{r}_i = 0 \quad \text{(principle of virtual work statics)}$$

this may seem trivial, but note that $\vec{F}_i^{ext} \neq 0$. What we've done is consider $\delta\vec{r}_i$ which *are consistent with constraints* such that $\sum_i \vec{F}^{ext} \cdot \delta\vec{r}_i = 0$. This is the *principle of virtual work for statics.* Virtual work is "virtual" in the sense that it is a construct where we consider virtual displacements of the system, not "real" displacements taking place in some time $t$, where $t$ would be determined by mechanics of the system.

### 1.3.2 Principle of Virtual Work for Dynamical System:

Now consider a dynamical system (here is where we introduce Newton's laws):

$$\vec{F}_i = \frac{d}{dt}\vec{p}_i$$

mathematically, this is just like statics if we introduce the intertial force $-\frac{d}{dt}\vec{p}_i$. This is called *D'Alembert's Principle.*

$$\sum \left( \vec{F}_i^{ext} - \frac{d}{dt}\vec{p}_i \right) \cdot \delta\vec{r}_i = 0$$

So far we have worked in cartesian coordinates. It is more convenient to transform to the generalized co-ordinates. Derive these transformations, *considering only holonomic constraints.*

$$\vec{r} = \vec{r}(\vec{q}, t) \quad \text{for holonomic constraints}$$

$$\vec{v} \equiv \frac{d\vec{r}}{dt} = \frac{\partial\vec{r}}{\partial t} + \sum_k \dot{q}_k \frac{\partial r}{\partial q_k} = \frac{d}{dt}\vec{r}$$

take the partial wrt $\dot{q}_k$

$$\frac{\partial \frac{d}{dt}\vec{r}}{\partial \dot{q}_k} = \frac{\partial \vec{r}}{\partial q_k} \quad \text{"dot cancellation"}$$

also

$$\frac{\partial \frac{d}{dt}\vec{r}}{\partial q_j} = \frac{\partial}{\partial t}\frac{\partial \vec{r}}{\partial \vec{q}_j} + \sum_k \dot{q}_k \frac{\partial^2 \vec{r}}{\partial q_k \partial q_j}$$
$$= \frac{d}{dt}\left(\frac{\partial \vec{r}}{\partial q_j}\right)$$

$\Rightarrow$

$$\frac{d}{dt}\vec{p}\cdot\delta\vec{r} = \sum_k m\frac{d^2}{dt^2}\vec{r}\cdot\frac{\partial \vec{r}}{\partial q_k}\delta q_k$$
$$= \sum_k m\left[\frac{d}{dt}\left(\frac{d\vec{r}}{dt}\cdot\frac{\partial \vec{r}}{\partial q_k}\right) - \frac{d\vec{r}}{dt}\cdot\frac{d}{dt}\left(\frac{\partial \vec{r}}{\partial q_k}\right)\right]\delta q_k$$
$$= \sum_k m\left[\frac{d}{dt}\left(\frac{d\vec{r}}{dt}\cdot\frac{\partial \frac{d\vec{r}}{dt}}{\partial \dot{q}_k}\right) - \frac{d\vec{r}}{dt}\cdot\frac{\partial \frac{d\vec{r}}{dt}}{\partial q_k}\right]\delta q_k$$
$$= \sum_k m\left[\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k}\right) - \frac{\partial T}{\partial q_k}\right]\delta q_k$$
$$where\ \ T = \frac{1}{2}m\frac{d\vec{r}}{dt}\cdot\frac{d\vec{r}}{dt}$$

if we have conservative force where $\vec{F} = -\frac{\partial V(\vec{r},t)}{\partial \vec{r}}$ then

$$\vec{F}\cdot\delta\vec{r} = -\frac{\partial V}{\partial \vec{r}}\cdot\sum_k\frac{\partial \vec{r}}{\partial q_k}\delta q_k = -\sum_k\frac{\partial V}{\partial q_k}\delta q_k$$

Hence, D'Alembert's principle gives

$$\vec{F}\cdot\delta\vec{r} - \frac{d}{dt}\vec{p}\cdot\delta\vec{r} = \sum_k\left[\frac{\partial V}{\partial q_k} - \frac{\partial T}{\partial q_k} + \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k}\right)\right]\delta q_k = 0$$

the virtual displacements of the generalized coordinates are by definition independent, so each term vanishes independently – note: this derivation is *dependent on having holonomic constraints.*

Introduce $L = T - V$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = 0 \quad \text{(Lagrange's Equations)}$$

one can argue back to Hamilton's principle from the Lagrange's eqns.

We have used Newton's laws directly to derive the Lagrange formalism, and we get the form of the Lagrangian ($L = T - V$) by doing so.

Why does T depend explicitly on $q_k$ (ie. $\frac{\partial T}{\partial q_k} \neq 0$)? In cartesian coordinates $T = \frac{1}{2}m\frac{d\vec{r}}{dt}\cdot\frac{d\vec{r}}{dt}$; and $\frac{\partial T}{\partial r_i} = 0$ .

To answer this question, consider motion in polar co-ordinates of a particle on a plane:

$$T \propto v^2 = \dot{r}^2 + r^2\dot{\phi}^2$$

$\frac{\partial T}{\partial r} \neq 0$ is case where co-ordinate system involves curved coordinates - ie. if constant generalized velocities result in curved motion of some parts of the system. $\frac{\partial T}{\partial r} \neq 0 \Rightarrow$"fictitious forces".

We have now a simple formalism using a simpler and more rational co-ordinate system – we can use it for *any* set of *independent* co-ordinates.

### 1.3.3 More on Forces

Consider now a system where forces are *not* derivable from the gradient of a potential.

In cartesian co-ordinates:

$$\vec{F}_{tot} = -\frac{\partial V}{\partial \vec{r}} + \vec{\mathcal{F}}$$

where the first term on the right contains forces derivable from a potential, and $\vec{\mathcal{F}}$ refers to other types of forces (e.g. friction).

Then we have:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k}\right) - \frac{\partial T}{\partial q_k} - \left(\frac{-\partial V}{\partial \vec{r}} + \vec{\mathcal{F}}\right) \cdot \frac{\partial \vec{r}}{\partial q_k} = 0$$

Incorporate the part derivable from a potential into the Lagrangian: $L = T - V$ as before

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = Q_k$$

$$Q_k = \sum_i \mathcal{F}_i \frac{\partial r_i}{\partial q_k}$$

*An important example: The Parachutist*

air resistance: $\vec{F} = -k\vec{v}$

$$Q_k = -k\frac{\partial \vec{r}}{\partial q_k} \cdot \frac{d\vec{r}}{dt} = -k\frac{\partial \frac{d\vec{r}}{dt}}{\partial \dot{q}_k} \cdot \frac{d\vec{r}}{dt}$$

introduce

$$\mathcal{F} = \frac{1}{2}kv^2 = \frac{1}{2}k\frac{d\vec{r}}{dt} \cdot \frac{d\vec{r}}{dt} \quad \text{Rayleigh dissipation function}$$

$$Q_k = -\frac{\partial \mathcal{F}}{\partial \dot{q}_k}$$

here $Q_k$ is the component of the generalized force due to friction – gravity is incorporated into $L$.

The Lagrange equations become:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} + \frac{\partial \mathcal{F}}{\partial \dot{q}_k} = 0$$

note: two scalar functions, $L$ and $\mathcal{F}$ must be specified to find the EOM.

We can also put the Lagrange equations in the familiar form

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = 0$$

if generalized forces are obtained from a function $U$ by

$$Q_j = -\frac{\partial U}{\partial q_j} + \frac{d}{dt}\left(\frac{\partial U}{\partial \dot{q}_j}\right)$$

in this case we can write $L = T - U$, $U$ is called a generalized or velocity-dependent potential. This applies to, for example, electromagnetic forces on moving charges. But note, this is now always the case (as with friction above).

## 1.4 Examples

*1. A particle moving in a conservative central force*

Newton: $m\ddot{x} = f\cos\phi$

$m\ddot{y} = f\sin\phi$

Lagrange: $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2) - V(r)$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = 0$$

Equation for r:

$$\frac{\partial L}{\partial \dot{r}} = m\dot{r} \quad ; \frac{\partial L}{\partial r} = mr\dot{\phi}^2 - \frac{\partial V}{\partial r}$$

$$m\ddot{r} - mr\dot{\phi}^2 - \frac{\partial V}{\partial r} = 0$$

Equation for $\phi$:

$$\frac{\partial L}{\partial \dot{\phi}} = mr^2\dot{\phi} \; ; \frac{\partial L}{\partial \phi} = 0$$

$$\frac{d}{dt}\left(mr^2\dot{\phi}\right) = 0 \quad \text{(a conservation law)}$$

$$mr^2\dot{\phi} = const \equiv l$$

so we get our EOM:

$$m\ddot{r} - \frac{l^2}{2mr^3} + \frac{\partial V}{\partial r} = 0$$

which you should recognize from the Newtonian approach.

*2. Let the particle be subject to a complicated elastic force which is still central*

Use generalized forces:

$$\begin{aligned} Q_r &= (f\cos\phi, f\sin\phi)\cdot\left(\frac{\partial x}{\partial r}, \frac{\partial y}{\partial r}\right) \\ &= f\cos\phi\frac{\partial x}{\partial r} + f\sin\phi\frac{\partial y}{\partial r} \\ &= f\left(\cos^2\phi + \sin^2\phi\right) = f \end{aligned}$$

$$
\begin{aligned}
Q_\phi &= f\cos\phi\frac{\partial x}{\partial\phi} + f\sin\phi\frac{\partial y}{\partial\phi} \\
&= -fr\cos\phi\sin\phi + fr\cos\phi\sin\phi = 0
\end{aligned}
$$

i.e.: $m\ddot{r} - mr\dot{\phi}^2 = f$ and $\frac{d}{dt}\left(mr^2\dot{\phi}\right) = 0$, and angular momentum is still conserved.

*3. Balls connected by a string, going through a hole in a table.*

$m_1$ and $m_2$ are connected by a string of length $l$, going through a hole in a table. $m_2$ can only move vertically. Choose for the generalized coordinates $r, \theta$.

$x_1 = r\cos\theta \quad y_1 = r\sin\theta \quad z_1 = 0$

$x_2 = y_2 = 0 \quad z_2 = r - l$

$$T = T_1 + T_2 = \frac{1}{2}m_1\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{1}{2}m_2\dot{r}^2$$

$$V = m_2 g\,(r - l)$$

$$L = T - V = \frac{1}{2}\left(m_1 + m_2\right)\dot{r}^2 + \frac{1}{2}m_1 r^2\dot{\theta}^2 - m_2 g r$$

$\theta$ again is easy:

$$\frac{d}{dt}\left(m_1 r^2\dot{\theta}\right) = 0 \rightarrow \text{ cons of angular momentum}$$

$r$ :

$$\frac{d}{dt}\left(\left(m_1 + m_2\right)\dot{r}\right) - m_1 r\dot{\theta}^2 + m_2 g = 0$$

again let $l = const = m_1 r^2\dot{\theta}$

$$\left(m_1 + m_2\right)\ddot{r} = \frac{l^2}{m_1 r^3} - m_2 g$$

Other examples we may do on the board if we have time: block on the inclined plane, cylinder rolling down an inclined plane, geodesic on a cone.

### 1.4.1 Interpretation:

Conceptually, there is a fundamental difference between Newton's laws and Hamilton's principle of least action. Newton's laws are a local description, while Hamilton's formulation depends on minimizing a function of the whole path. If you have a principle of this kind, you should get a local description if you consider small paths. We'll show next week that this is the case.