Physics 106a: Classical Mechanics

Homework 3: Applications of Lagrangian Dynamics

Due: Thursday, October 21, 1999

Recommended reading: Goldstein pp. 45 – 63
Supplemental reading: Landau and Lifshitz 25 – 27, 80 – 83

1. (Pendulum in an accelerated reference frame.) A pendulum with a weightless string of length $D$ and mass $m$ is attached to a moving car. The car is continuously accelerated along a horizontal track with constant acceleration $a$, starting from an initial horizontal velocity $v_0$. Gravity acts in the vertical direction.

Assume that the $(x,y)$ co-ordinate system is at rest with respect to the ground. Use the angle between the vertical and the pendulum bob, $\theta$, as the generalized co-ordinate.

a.) Find the components of the velocity of the pendulum bob in the laboratory frame (the frame at rest with respect to the moving car). Find the kinetic energy as a function of $\theta$, $\dot{\theta}$, and the other variables (all of which are known functions).

b.) Find the Lagrangian $L(\theta, \dot{\theta}, t)$. Does $L$ depend explicitly on time?

c.) Find the equation of motion for the pendulum.

2. (Lagrangian, Energy Integrals.) A flyball governer (for regulating motor speed) is comprised of two balls of mass $m_1$ attached by light struts of length $a$ to a fixed bearing $A$, and by another pair of struts, also of length $a$ to a mass $m_2$ that can slide freely on a vertical axle. The governer rotates about this axle with fixed angular velocity $\omega$.

\[ \theta m_1 \quad m_2 \]

\[ m_1 \quad \theta \quad \theta \quad a \]

\[ a \quad \theta \quad \omega \quad a \]

\[ m_1 \]

a.) Choosing the angle $\theta$ that a strut makes with the vertical as a coordinate, write down a Lagrangian.

b.) Use your Lagrangian to determine how $\theta$ varies with $\omega$ in an equilibrium solution.

c.) Find an energy integral.

d.) Find the frequency of small oscillations about the equilibrium.

3. (Period of a plane pendulum.) Show that for small $m$

\[ K(m) = \frac{\pi}{2}(1 + m/4 + 9m^2/64 + ...) \]  

(1)

Thus, if a pendulum clock beats seconds when its maximum swing is $10^\circ$, how many minutes per day will it lose if its amplitude is increased to $20^\circ$?

Extra Credit. Consider the full non-linear equations that we derived in class for the double pendulum. Write a computer program to solve them numerically, and explore solutions for large amplitude motion. Are they always periodic?